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The Authors

Glossary

Keywords
1 Introduction

Ingo Arne Hansen

Timetabling and traffic management are two basic elements of public transport operations, which are inherently linked. The function of the timetable in practice, however, is often only loosely connected to its execution. Customers, drivers, conductors, and dispatchers interpret the timetable differently and depend on specific sources of information and technical support. Further, the feasibility of railway schedules and operations is subordinated to the constraints of the signalling and safety systems. For trip planning and decision making, railway passengers used timetable books and displays at stations, supported by telephone advice, personal travel information assistance and ticket sales. In recent years, live travel information and booking via the internet have become more and more available.

In many railways, however, train drivers and conductors still receive only a printed daily schedule indicating the relevant departure, passing and arrival times, as well as the times and locations for the start, change and end of their duties, and a train radio or mobile phone for communication with the traffic control centres. Electronic on-board timetable information in the driver’s cab and in the passenger coaches has, so far, been introduced only by a limited number of railways. During train operations, drivers must observe the signals and cab displays, which indicate the necessary safety information with respect to movement authority, permitted maximum speed, and in some modern trains the critical target distance. It is perhaps surprising that continuously updated actual delay information for train drivers is still generally missing, although standard on-board information displays were introduced on buses, tramways and light rail lines more than 20 years ago. A more precise and actual traffic information feedback to drivers and conductors would help them to better recognise deviations from schedules, to anticipate conflicts, and to raise operational performance.

The information made available to signallers, dispatchers and network supervisors in traffic control centres has improved significantly. This has been due to the introduction of first, relays, and then electronic interlocking, associated with wall-based panoramic displays followed by computer screen monitoring of the actual set-up of routes, track occupancy and sectional clearances for each train. Further, time-distance diagrams of the daily scheduled and the realised paths became available at the newly built control centres. These are equipped with modern information, telecommunications and computer technology.

Advanced tools for timetable design and construction have been implemented in the past two decades. These enable a much higher quality of timetabling and contribute to more reliable operations. However, the punctuality levels achieved by railways in general don’t give evidence of higher performance – except from Japan. It is hypothesised that the disappointing train punctuality performance of many networks stems only partly from the high traffic volumes and capacity usage. The main cause is an insufficient degree of optimisation, exactness and robustness of the current practice of timetabling, which misses the feedback of detailed operational performance data.

This book intends to narrow the gap between theory and practice of railway timetabling and traffic management by compiling the current state-of-the-art in this field and indicating a number of remaining problems and research issues. It is directed at academics, Masters and PhD students, professionals from the railway industry, and also public authorities that tender and supervise railway services. Its aim is to enable readers to better understand the underlying theoretical models and to become acquainted with the actual state of technology.

The authors of the individual chapters are members of the International Association of Railway Operations Research (IAROR)\(^1\), founded in 2004, and were selected on the basis of expertise in their

\(^1\) see: www.iaror.org
specific areas. All are involved currently in railway research projects and have published individually in journals or international conference proceedings. The book forms a compendium of the main methodological aspects of railway timetabling, traffic modelling and analysis, including some examples of calculation and practical application.

The book is structured around the elementary steps of railway timetable design, modelling of infrastructure and train operations, and the analysis, optimisation, simulation, and evaluation of timetables and traffic respectively. In Chapter 2, Pachl outlines the principles of timetable design and explains the methods for estimating the constituent diagrams and variables as a function of different signalling systems. In the following Chapter 3, Radtke deals with the models for description of the infrastructure based on graph theory, while showing the differences between node and link oriented, as well as macroscopic and microscopic models.

Chapter 4 contains a detailed explication of the calculation of train running times by Brünger & Dahlhaus, depending on infrastructure and rolling stock characteristics, followed by a discussion of standard and advanced deterministic and stochastic calculation methods. Albrecht continues in Chapter 5 by describing the practice of energy supply and billing, and then explains the fundamentals and an intelligent tool for energy-efficient train control and driving, depending on the actual state of operations.

The following three Chapters 6, 7 & 8 focus on mathematical models for the estimation of waiting times, delays, delay propagation in networks, and optimal train scheduling. Wende describes the fundamentals of queueing theory and its application, while Goverde presents a Petri net approach by means of the so-called max-plus algebra technique for stability analysis of large network timetables. Kroon, Huisman and Maróti describe how optimisation models and techniques can be used to generate cyclic timetables and routes through stations in an automatic way.

Chapter 9 is devoted by Siefer to a description of different simulation models for timetable design and train operations focusing on microscopic synchronous approaches. In the next Chapter 10, Yuan explains the application of a standard and a new engineering approach for the detailed statistical analysis of train delays based on standard track occupation and release data, including the methods to test the goodness-of-fit and to assess the distributions and its parameters.

Chapter 11 consists of a review of the existing models for the rescheduling of trains in case of service disruptions. Jacobs explains the architecture and required modules of computerised decision support systems for dispatchers. In Chapter 12, the available methods for the evaluation of performance of railway operations, networks and enterprises are discussed by Martin. Finally, the Editors of this book, Hansen and Pachl, derive the main conclusions concerning the current state-of-knowledge on railway timetabling and traffic modelling and analysis from the preceding Chapters. They then summarise the remaining research topics. Their aim is to encourage readers of the book to contribute further to a better comprehension of railway processes and to improve the attractiveness and efficiency of train services for railway customers.

The approaches presented are not purely representative of the theories and tools developed in universities and railway undertakings world-wide. The book is, in the first instance, a German-Dutch co-production, where the academic and professional quality of research and education, and the application of railway technology, are amongst the best in the world. The Editors hope that researchers from other countries will enjoy the content as well and use it for improving the analysis and modelling of railway timetabling and operations on their own.
3 Infrastructure Modelling

In Fig. 3.13 a benchmark for the levels of importance of the different steps between microscopic and macroscopic models can be seen. Macroscopic models hold advantages in respect of the first four criteria. However, applying an established and well-maintained microscopic database those criteria may not be the most dominant points. The last criterion, “accuracy of results”, is much more important when compared against all other criteria. From previous experience, the value of correct results during the planning process should be highlighted.

Microscopic infrastructure models have been used successfully in various countries around the world. In countries like Germany, Austria and Switzerland, microscopic infrastructure network models have been used since the beginning of the 1990’s. Other railway companies or government bodies (e.g. the European Union) should be encouraged to follow this approach in the future as well.

3.8 Outlook

The future of the utilisation of railway infrastructure models will probably include the following three trends:
- Enhancement of railway infrastructure models with further attributes and functionality. Currently some research work is going to establish a link between railway planning data with cost-orientated asset data. This includes the consideration of Life Cycle Costs of infrastructure asset elements and a financial evaluation of infrastructure and train operation in respect to budget spending.
- The developing of standard interface formats may help to exchange infrastructure data across railway undertakings or to third parties. However, due to the complexity of railway infrastructure models in comparison with timetable exchange formats, there are serious doubts that a “standard” infrastructure format will be established in the near future. For timetable information, an exchange format like RailML or other XML-based formats could be a reasonable approach. Furthermore, SQL can be used to communicate with databases.

Fig. 3.14: Train route interface shown with Google Earth v4®
– Linking railway planning infrastructure data with Geographical Information System (GIS) data. Railway planning systems should be able to support all common graphic formats (BMP, JPEG, TIFF, etc.) and to set multiple images with individual scale and position functionality. The link between both modelling approaches will enable planners to better present complex results to decision makers and the public. Fig. 3.14 illustrates a simple example showing a train run in a Google Earth map of Austria.

– Generating infrastructure models from public sources. These technologies may be used by third parties who do not have – for any reason – access to the railway undertaking infrastructure databases:
  – Network statements of railway undertakings from the internet
  – Electronic maps
  – Internet maps (e.g. Google Earth® or Virtual Earth of Microsoft®)
  – Printed public maps
  – GPS and video technology.

– Establishing Corporate Network Models (CNM) through the integration of railway planning data with asset management data, geographical information, as well as data from the engineering, control and other departments.

The microscopic infrastructure for railway planning will be useful as an initial step in the creation of the CNM. The main challenge at present is to maintain the microscopic infrastructure model for purposes such as running time calculation and timetable planning.

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4 Running Time Estimation

Olaf Brünger, Elias Dahlhaus

4.1 Introduction

4.1.1 Fundamentals

Trains are run to a schedule, so why is there a need to estimate their running time? The physical process of accelerating a train is quite approximate in the time it takes, depending as it does on the friction between wheel and rail, the overhead line voltage, and the actions of the train driver. But it is even necessary to calculate running times for very short distances using precise numerical calculations (with high attention afforded to the exactness of the arithmetic and counting the seconds).

When setting up a timetable, it is necessary to estimate running times for different train configurations using the current infrastructure, which is normally different from last year's infrastructure or even changing within the timetable period. Examining capacity of nodes and routes, constraints like minimum headways are considered – which are derived from the speed and time when the trains approach and clear the critical block signals and track sections. Normally, these times are not given with the timetable, so they have to be calculated. A linear interpolation derived from departure and arrival times is wrong, so it has to be recalculated based on how the train accelerates, runs, and brakes. Accurate running times are used to achieve good dispatching results or to save energy by using margins in the timetable.

Most modern tools for analysis or timetable construction calculate running times on demand for the train concerned. In more simple circumstances, it is enough to prepare tables of running times or headways for certain train configurations and for different combinations of stopping and passing stations. (Note that in preparing these tables, very detailed infrastructure and train data are also needed!)

Many influences on running times are not deterministic, such as human behaviour, weather, train composition, infrastructure works, the rolling stock, and route condition. To cover this, random influences are taken into consideration through recovery times, and the basic calculation is done with standard values representing good conditions. Another approach is to use probabilistic approaches as described in a later section of this Chapter.

However, some information is fundamental for estimating running times:
– route details with distance, speed limits, and gradient (and curves and tunnels)
– traction unit characteristics, with tractive effort and resistance
– rolling stock characteristics, with mass, length, and resistance (mainly dependent on the kind of train)
– given data of the operating cycle such as starting point, stops, and – as appropriate – timetable restrictions.

Dependent on safety devices, information about brake effort or braking supervision might also be necessary.

Historically, running times were estimated using manual graphic algorithms or using mechanical instruments like a Conzen-Ott-machine (a mechanical analog integrator for running time estimation) as shown in figure 4.1, which was already capable of taking these values into account. This book concentrates on the main topics of running time estimation using computers. Reference may be made to special literature as (Wende, 2003) for the more detailed aspects.
4.1 Introduction

4.1.2 Speed Profile

The speed profile of a train on a given route is often visualised in a speed-distance-diagram. It is useful to add a gradient-distance diagram to see the main line influence. The authors examine a typical speed profile as shown in figure 4.2, where the principal order of acceleration, constant movement, braking, and stop can be seen.

Fig. 4.1: Conzen-Ott-Machine for Estimating Running Times

Fig. 4.2: Sample Speed Profile

1. The horizontal axis shows the route measured in kilometres. Both diagrams have identical divisions.
2. The vertical axis of the speed-distance-diagram shows the velocity measured in km per hour (km/h) and metres per second (m/s). Both the allowed and actual speeds are shown in the same diagram, which is concerned with the front of the train.
3. The vertical axis of the second diagram shows the gradient of the line measured per 1,000 – positive values for ascents, negative for descents. Note that railway gradient angles are very slight, so it makes only a negligible difference whether the horizontal measurement is truly so, or along the length of the track (tan α = sin α).
4 Running Time Estimation

4. Maximum speed of the train is 160 km/h.
5. Speed limit given by infrastructure constraints changes several times on the route. It starts with 100 km/h and rises later to 140 and to 200 km/h.
6. At km 15 there is a station where entry speed is reduced to 60 km/h which is often caused by points. Note that the difference in running time estimation caused by turnout speed is dependent on the track used by the train. In a detailed analysis or timetable construction this difference has to be considered, as it can easily reach a minute, or more.
7. Exit speed from the station and track is reduced to 40 km/h. Later, the train is allowed to run at 100 km/h again.
8. At km 28 there is a special restriction to 80 km/h. There are many possible reasons: Work sites with speed restrictions (for all trains), defects in contact wire (for electric trains only), tunnels (often for passenger trains only), wind danger (for empty freight trains). Also, special increases of the speed limit for several trains exist, for example for tilting trains. When calculating very precisely, there are also dependencies between different trains, so that running time estimation is not for a single train only. For example, if the meeting of trains in a tunnel is restricted to a certain (relative) speed limit, or if the exit speed of a train from a station is dependent on the preceding train.
9. In the other diagram, two small descents of 5 and 8 per 1,000 can be seen. Dependent on the braking efforts of the train and the gradients characteristics of the line, there might be special speed reductions.
10. There are also two ascents, one of which is quite steep (40 ‰, a typical value for suburban train operations).
11. Considering the estimated speed of the train now: At first, it accelerates from 0 until it reaches the actual speed limit of 100 km/h.
12. It remains at the speed limit by constant movement.
13. When the speed limit rises, it accelerates again to the new speed limit of 140 km/h. Note that it starts accelerating a little later: Speed limits refer to the whole train, and the head of the train is marked. The train driver (and the simulating computer model) has to wait with his acceleration until the back of the train has passed the point.
15. Acceleration up to the speed limit of this train which cannot make full use of the speed allowed for the line.
17. The train starts braking. The position is dependent on the following.
18. At the point where the entry speed restriction takes place, the train must already be running at the lower speed. The train driver (and the model) has to estimate where to start braking to achieve this. This is done by calculating the position of intersection (no. 17) between the braking curve and that of the constant movement beforehand. Of course in practice this does not always occur exactly, but as mentioned before good conditions are assumed when estimating running times (and recovery times added subsequently).
19. Constant movement (in the entry section of the station).
20. Braking to stop. (When estimating complete scheduled times used by the train, the dwell time has to be added here.)
21. Acceleration to the exit speed.
22. Constant movement.
23. Acceleration to the speed limit of the line, then constant movement again.
24. On the uphill gradient, the train loses velocity: The effort of the train does not compensate fully for the sum of resistances, including the line resistance of the steep incline. As will be seen later, the tractive effort increases as speed reduces, and resistances have the opposite behav-
4.1 Introduction

There may be a velocity where the train moves at constant speed as in the example; otherwise, it would reduce speed until it stops on the incline.

25. When the top of the hill has been reached, the train accelerates again. Note that the effect does not start with the head of the train but a bit later: In a very detailed calculation the train is regarded as a (homogeneous) strip, whereas normally the impact of the gradient is just shifted by half of the train length.

26. Because the special speed limit is valid for this train, it has to reduce its speed by braking. It continues by constant movement and acceleration later.

This diagram shows no coasting (run-out) section of the train movement because the calculation is for the shortest running times in good conditions and recovery times are added later. Of course in practice the train driver will make use of reserves by reducing power (while the resistances continue) before he has to brake for a stop or a speed limit reduction. In a computer model (like that implemented in the electronic display of energy saving driving in trains of Deutsche Bahn), the energy-optimal position of starting run-out can be calculated dependent on the amount of recovery time left. Chapter 5 deals with this topic.

4.2 Infrastructure and Train Data

In the last section it was seen in principle how the train behaves on a given route. To calculate running times in the different sections, it is now necessary to look at the influences of infrastructure and train, especially for calculating the acceleration section of the speed profile. It is useful to look at four different components:

– tractive effort \( F_T \),
– traction unit resistance \( F_{Rt} \),
– rail vehicle or wagon resistance \( F_{Rw} \),
– infrastructure data, especially line resistances \( F_{Rl} \).

The difference between tractive effort and the sum of the resistances is the effort available for accelerating the train.

Note: the following formulas use the physical quantity with the SI-dimensions, including

– velocity \( v \) measured in [m/s] (3.6 \( v \) is the value in [km/h]) and
– mass \( m \) measured in [kg].

The quantity \( g \) is the earth gravity constant 9.81 m/s\(^2\).

4.2.1 Tractive Effort \( F_T \)

The locomotive or the power equipment of the multiple unit generates effort which is intended to move the train. It is called induced tractive effort \( F_{Ti} [N] \).

However, not the whole amount of this effort can be used, due to three principle reasons:

– The internal power transmission of the locomotive or the multiple unit consumes between 2\% and 3\% of the effort,
– the effort can be limited to a constant maximum amount to prevent the power equipment from overheating,
– the wheels will spin if the effort exceeds the maximum adhesion between wheel rim and rail.

The last topic is described by the adhesion value \( \mu \) (a factor without dimension, dependent on driven speed \( v \) [m/s]) and the wheel load \( F_L [N] \) of the driven wheels. The product of both is the maximum effort which is transmitted without spinning wheels. In first